DEVELOPMENT OF THE POLAK MODEL

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INTRODUCTION

Macroeconomic stability is very important for each economy because it constitutes the basis of sustainable economic growth and development. It means stable prices with a low level of inflation, a stable foreign exchange rate and balance of payments.

The most important institution that provides financial support to countries which have problems with internal and external stability is the International Monetary Fund (IMF). Credits are issued after an analysis of the causes of internal and external deficits. The IMF approach to macroeconomic stabilization is based on a monetary approach to the balance of payments. The first IMF model designed for dealing with balance of payments disequilibrium was the Polak model on monetary programming. This model yields a policy package that must be implemented by countries receiving support. In other words, the Polak model is used as the foundation of the IMF’s credit arrangements.

The model can be modified in various ways. The aim of this paper is to show the main modifications of the Polak model and their implications for the relevant fiscal and monetary policies.

1. THE POLAK MODEL

J.J. Polak presented his model in 1957 in a paper titled Monetary Analysis of Income Formation and Payments Problems. The model indicates what macroeconomics policies are required to achieve a given set of outcomes. In other words, it determines policy targets consistent with explicit macroeconomic objectives.

The standard Polak model is simple, with a very limited number of variables. It was designed to be usable in every developing country. It is therefore very compact, with minimum data requirements. The model needs

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2 Two separate versions of the 1957 model for developed and developing countries were presented in J.J. Polak and V. Argy, ‘Credit Policy and the Balance of Payments’ (1971) 16 IMF Staff Papers 1-24.
as inputs only banking data and trade data. These sets of statistics are generally available in all developing countries. Moreover, it can be flexibly applied and explained with relative ease to policy makers with no experience of empirical modelling.\(^3\)

The Polak model integrates monetary, income and balance of payments analyses,\(^4\) and consists of a set of four equations. The model contains two behavioural relationships: the demand for money function and the function of the demand for imports, and two identities: for the money supply and for the balance of payments.

The model assumes the simplest form of the demand for money function. The amount of money that people want to hold is proportional to nominal income by a factor which is the inverse of the income velocity of money. The money market is in equilibrium if the demand for money is equal to the amount of money issued by the monetary and banking sectors (the supply for money). As a result, the following equation is satisfied:

\[
MS_t = \frac{1}{\nu} Y_t, \quad \nu > 0, 
\]

where \(MS_t\) is the money supply, \(Y_t\) is nominal income in time \(t\), and \(\nu\) is the income velocity of money (the average frequency with which a unit of money is spent in a given period of time).

Under the condition that the income velocity of money is constant, the change in the money supply is given by

\[
\Delta MS_t = MS_t - MS_{t-1} = \frac{1}{\nu}(Y_t - Y_{t-1}) = \frac{1}{\nu} \Delta Y_t.
\]

The equation (2) is often written in the form

\[
\Delta MS_t = k \cdot \Delta Y_t,
\]

where \(k = 1/\nu\) is the coefficient of the demand for money. The equation (3) yields a constant relationship between the change in the money supply and the change in nominal income. Any increase in nominal income is equal to the increase in the money supply times the income velocity of money.

The demand for imports depends on the level of a country’s nominal income

\[
M_t = m \cdot Y_t, \quad m \in (0,1),
\]

where \(M_t\) is imports of goods and services in time \(t\), and \(m\) is the constant marginal propensity to import (the ratio of the change in imports to the change in nominal income). The equation (4) implies that an increase in nominal income leads to a proportional increase in imports. From (3) and (4) follows that the model has a dynamic character. It contains both nominal income and change in nominal income.

The money supply is equal to the domestic credit of the banking system and the value of net foreign currency reserves. The change in money supply is by definition equal to the change in a country’s foreign currency reserves and the change in domestic credit of the banking system:

\[
\Delta MS_t = \Delta R_t + \Delta DC_t,
\]

\(^3\) FAG Den Butter, MS Morgan, ‘What makes the models-policy interaction successful?’ (1998) 15 Economic Modelling 468.

where $\Delta R_t$ is the change in foreign reserves and $\Delta DC_t$ is the change in domestic credit in time $t$. Domestic credit expansion is taken as an independent variable. In other words, a credit to the nonbank sector is an exogenous policy variable under the control of the monetary authorities.

The change in foreign reserves is by definition equal to net exports and net capital inflows of the nonbank sector:

$$\Delta R_t = X_t - M_t + K_t,$$

where $X_t$ is exports of goods and services, $M_t$ as in equation (4), and $K_t$ is the net capital inflows of the nonbank sector. The value of exports and the value of net capital inflows are independent (exogenous) variables. The value of imports is a dependent (endogenous) variable. Equation (7) identifies components of the balance of payments.

Using equations (3) and (6), it is possible to determine the effects of changes in domestic credit on foreign currency reserves:

$$\Delta R_t = \frac{1}{\nu} \Delta Y_t - \Delta DC_t,$$

where all variables remain as before. Reserves will be stable if the change in domestic credit is equal to the change in nominal money demand, which depends in turn on the change in nominal income. Reserves will decline if an increase in domestic credit exceeds an increase in nominal money demand. One can estimate the required change in domestic credit for a given level of change in nominal income and a target level of foreign currency reserves. Equation (8) describes the money-market equilibrium condition. Graphically it can be presented by a positively-sloped straight line $MM$ in the $\Delta R_t - \Delta Y_t$ space (Fig. 1).

From equations (7) and (4) one obtains the balance of payments identity:

$$\Delta R_t = X_t - m(\Delta Y_t + Y_{t-1}) + K_t,$$

or in the form

$$\Delta R_t = -m \Delta Y_t - mY_{t-1} + X_t + K_t,$$

where all variables remain as before. Equation (10) is a negatively-sloped straight line $BP$ in the $\Delta R_t - \Delta Y_t$ space (Fig. 1).

Figure 1. The Polak model
The intersection of the line \( BP \) described by eq. (10) and the line \( MM \) (8) gives equilibrium values of the balance of payments and the change in nominal income (the equilibrium point \( E \) in Fig. 1). The increase in the rate of expansion of domestic credit (a shift of the line \( MM \) to the line \( MM' \)) causes the balance of payments to deteriorate and nominal income to rise (the new equilibrium point \( E' \)). In the short run an increase in domestic credit expands the money supply. According to the quantity theory of money, nominal income thus rises as well. For a given exports of goods and services and net capital flows an increase in nominal income causes an increase in imports of goods and services. As a result, foreign currency reserves fall and the balance of payments deteriorates. An increase in receipts of foreign currencies improves the balance of payments and raises nominal income (a shift of the line \( BP \) to the line \( BP' \)).

The model can be solved for a change in the level of foreign currency reserves and in nominal income. In the short term one obtains:

\[
\Delta R_t = \frac{1}{1 + m \cdot v} \left( \Delta Y_t - \Delta K_t \right) - \frac{m \cdot v}{1 + m \cdot v} \Delta DC_t + \frac{1}{1 + m \cdot v} \Delta R_{t-1},
\]

and

\[
\Delta Y_t = \frac{v}{1 + m \cdot v} \left( \Delta X_t + \Delta K_t \right) + \frac{v}{1 + m \cdot v} \Delta DC_t + \frac{v}{1 + m \cdot v} \Delta R_{t-1},
\]

where all variables remain as before.

The change in foreign currency reserves and the change in nominal income depend on the values of current and past years of exports of goods and services, capital inflows of the nonbank sector and the change in domestic credit of the banking system.

The long-run solution of the Polak model has the following form:

\[
\Delta R_t = \frac{1}{m \cdot v} \left( \Delta X_t + \Delta K_t \right) - \Delta DC_t
\]

and

\[
\Delta Y_t = \frac{1}{m} \left( \Delta X_t + \Delta K_t \right).
\]

In the long-run it is assumed that \( \Delta R_t = \Delta R_{t-1} \). Equations (13) and (14) are called the “reduced form” of the Polak model.

This simple form of the model was useful in the postwar years when fixed exchange rates dominated and balance of payments problems among IMF members were due to bursts of excessive domestic expansion.\(^5\)

The more complex version of the model distinguishes between nominal and real output and sources of domestic credit.

2. THE EXTENDED POLAK MODEL

In the extended version of the Polak model nominal income is regarded as an endogenous variable. Therefore it can be expressed in the form

\[
Y_t = P_t \cdot y_t
\]

where $y_t$ is real output that is regarded as an exogenous variable and $P_t$ denotes the overall price index.

The change in nominal output can be calculated as

$$\Delta Y_t = Y_t - Y_{t-1} = P_t \cdot y_t - P_{t-1} \cdot y_{t-1}.$$  

The right side of the equation (16) has the form

$$\Delta Y_t = \Delta P_t \cdot y_{t-1} + P_{t-1} \cdot \Delta y_t,$$

when $\Delta P_t$ and $\Delta y_t$ are so small that the second-order term $\Delta P_t \cdot \Delta y_t$ can be neglected.

In equation (17) the variables $y_{t-1}$ and $P_{t-1}$ are predetermined, the change in real output is exogenous and the change in price is endogenous.

Other changes in the model concerns credit creation, which is split into credit to the private sector $DC^p_t$ and credit to the government sector $DC^g_t$.

$\Delta DC_t = \Delta DC_t^p + \Delta DC_t^g.$

In the extended Polak model a change in foreign currency reserves can be presented as the following function:

$$\Delta R_t = \frac{1}{V} \cdot y_{t-1} \cdot \Delta P_t + \frac{1}{V} \cdot P_{t-1} \cdot \Delta y_t - \left( \Delta DC_t^p + \Delta DC_t^g \right).$$

In order to receive equation (19) one needs to insert equations (17) and (18) into (8). Equation (19) describes the money-market equilibrium condition. Graphically it can be presented by the positively-sloped straight line $MM$ in the $\Delta R_t - \Delta P_t$ space (Fig. 2). Equation (19) contains two endogenous variables, $\Delta R_t$ and $\Delta P_t$. However, it is not possible to find a unique solution for both these variables for a given expansion of domestic credit.

The balance-of-payments identity in the model has the form

$$\Delta R_t = -m \cdot y_{t-1} \cdot \Delta P_t + X_t + K_t - m \cdot (Y_{t-1} + P_{t-1} \cdot \Delta y_t).$$

Equation (20) is obtained from (7) when one assumes that

$$M_t = m(Y_{t-1} + P_{t-1} \cdot \Delta y_t + y_{t-1} \cdot \Delta P_t).$$

The relationship between reserves and changes in price levels (inflation) can be graphically presented by the negatively-sloped straight line $BP$ in the $\Delta R_t - \Delta P_t$ space (Fig. 2).
The intersection of the line $BP$ described by equation (20) and the line $MM$ described by equation (19) yields the equilibrium levels of the balance of payments ($\Delta R^*$) and inflation ($\Delta P^*$) at point $E$.

Changes in domestic credit can move the equilibrium point only along the line $BP$. For example, a reduction in the expansion of domestic credit shifts the line $MM$ upwards to the line $MM'$. At the new equilibrium point $E'$ reserves are higher at lower values of inflation. However, targets for reserves and inflation different from those along the line $BP$ (such as $E_i$) cannot be achieved only through changes in domestic credit. In other words, targets for reserves and inflation cannot be chosen independently. In order to achieve two targets we need two policy instruments. As a second policy tool the nominal exchange rate is chosen.

Using a new variable the change in price levels can be expressed as

$$
\Delta P_t = (1 - \theta) \cdot \Delta P_{d,t} + \theta \cdot (\Delta e_t + \Delta P_{m,t}),
\quad \theta \in (0, 1),
$$

where $P_{d,t}$ is the index of domestic prices in time $t$, $\theta$ is the share of imported goods and services in the general price index, $P_{m,t}$ is the price of imported goods and services measured in foreign currency and $e_t$ is the level of the nominal exchange rate. In equation (22) the change $\Delta P_{m,t}$ measures foreign inflation and $\Delta P_{d,t}$ is an additional endogenous variable. Domestic inflation depends on weighted domestic price changes and foreign price changes.

If one assumes that foreign inflation is negligible and equals zero then

$$
\Delta P_t = (1 - \theta) \cdot \Delta P_{d,t} + \theta \cdot \Delta e_t.
$$

Including the exchange rate as a policy variable requires a modification of equation (6) in order to take into consideration the impact of exchange rate changes on the balance sheet of the central bank:

$$
\Delta M_{S,t} = \Delta R_t + R_{f,t-1} \cdot \Delta e_t + \Delta DC^{e}_t + \Delta DC^{p}_t,
$$

where $R_{f,t-1}$ is the foreign currency value of the previously existing stock of foreign currency reserves.  

Substituting (2), (17) and (22) into (24) one receives

$$
\Delta R_t = \frac{1 - \theta}{v} \cdot y_{t-1} \cdot \Delta P_{d,t} + \left( \frac{\theta}{v} \cdot y_{t-1} - R_{f,t-1} \right) \Delta e_t + \frac{\theta}{v} \cdot y_{t-1} \cdot \Delta P_{m,t}
$$

$$
+ \frac{1}{v} \cdot P_{t-1} \cdot \Delta y_t \left( \Delta DC^{p}_t + \Delta DC^{e}_t \right)
$$

The modified money-market equilibrium condition (25) can be illustrated by the positively-sloped straight line $MM$ in the $\Delta R_t - \Delta P_{d,t}$ space.

In order to obtain the modified form of the balance-of-payments identity, imports in nominal terms should be described as a product of import volume and the nominal exchange rate:

$$
M_t = e_t \cdot Q_{m,t}, \quad \text{with } P_{m,t} = 1,
$$

where $M_t$ is imports in nominal terms, $Q_{m,t}$ is the import volume and $P_{m,t}$ is the foreign currency price of importable goods. Equation (26) yields

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\[ \Delta M_t = e_{t-1} \cdot \Delta Q_{m,t} + Q_{m,t-1} \cdot \Delta \epsilon_t. \]

Changes in import volume depend on the change in real income and the relative price of domestic and foreign goods as follows:
\[ \Delta Q_{m,t} = m \cdot \Delta y_t + \eta \left( \Delta P_{d,t} - \Delta e_t - \Delta P_{m,t} \right), \]
where \( \eta > 0 \) measures the elasticity of import volumes to changes in relative prices of imported goods and services. An increase in real output or an increase in domestic prices leads to increased imports. A devaluation in the nominal exchange rate, \( \Delta e_t > 0 \), will lower the changes in the volume of imports.

Substituting equation (28) into (27) one obtains
\[ M_t = M_{t-1} + (Q_{m,t-1} - \eta \cdot e_{t-1}) \cdot \Delta e_t + e_{t-1} \cdot (m \cdot \Delta y_t + \eta \cdot (\Delta P_{d,t} - \Delta P_{m,t})). \]

With a relatively small \( Q_{m,t-1} \), a devaluation in the nominal exchange rate improves the trade balance and increases foreign currency reserves.

Finally, inserting (29) into equation (7) and using the foreign currency value of exports and net foreign capital inflows, one gets
\[ \Delta R_t = -\eta \cdot e_{t-1} \cdot \Delta P_{d,t} - M_{t-1} + (X_t + K_t - Q_{m,t-1} + \eta \cdot e_{t-1}) \cdot \Delta e_t \]
\[ + (X_t + K_t - m \cdot \Delta y_t + \eta \cdot \Delta P_{m,t}) \cdot e_{t-1}. \]

Equation (30) can be graphically presented by the negatively-sloped straight line \( BP \) in the \( \Delta R_t - \Delta P_{d,t} \) space. The behaviour of the extended Polak model can be analysed in the same way as in Figure 2, except that the variable on the horizontal axis is now the change in domestic prices \( \Delta P_{d,t} \).

The point \( E_1 \) can be attained when the authorities exert control over \( \Delta e_t \).

Equations (25) and (30) can be simultaneously solved to obtain values for the endogenous variables of foreign currency reserves and inflation.

In the extended Polak model the authorities have two policy instruments at their disposal. They can attain the targeted values for both the balance of payments and the rate of inflation.

The main differences between the standard and extended Polak model are presented in Table 1.

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\(^8\) Alternatively, from equations (23) and (28) the change in domestic credit and the change in nominal exchange rate (two policy instruments) can be obtained for given target values for foreign reserves and inflation.

\(^9\) Khan, Montiel, Haque (n 6) 19.
Table 1. The structure of the IMF model on financial programming

<table>
<thead>
<tr>
<th>Targets</th>
<th>Endogenous variables</th>
<th>Exogenous variables</th>
<th>Policy variables</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>The standard Polak model</td>
<td>foreign currency reserves</td>
<td>stock of money, imports of goods and services, nominal income, exports of goods and services, net capital inflows of the nonbank sector</td>
<td>domestic credit</td>
<td>the income velocity of money, the marginal propensity to import</td>
</tr>
<tr>
<td>foreign currency reserves, domestic prices</td>
<td>nominal income, stock of money, imports of goods and services</td>
<td>real income, foreign prices, exports of goods and services, net capital inflows of the nonbank sector</td>
<td>Domestic credit, nominal exchange rate</td>
<td>income velocity of money, marginal propensity to import, share of imported goods and services in the price index, price elasticity of imports</td>
</tr>
</tbody>
</table>

CONCLUSIONS

The Polak model is the basis of the IMF’s stabilization program, which concentrates mainly on the relationship between internal and external stability. According to the standard model, control over net domestic credit expansion is the key to stabilizing the level of foreign currency reserves and therefore the balance of payments. The excess growth of net domestic credit over growth in money demand yields the current account deficit. If the money supply exceeds money demand then people attempt to get rid of additional money. They buy domestic or foreign goods, causing an increase in the level of inflation and imports. A country cannot at the same time reverse a balance of payments deficit and increase domestic credit. The model shows how to calculate a rate of domestic credit creation consistent with a target for improvement in the balance of payments.

The extended Polak model gives quantitative methods for determining targets for the level of inflation and the level of foreign currency reserves that are consistent with macroeconomic stability.